

B. Loiseau · J.-P. Dedonder · A. Furman ·  
 R. Kamiński · L. Leśniak

# *CP* violation and strong pion-pion interactions in the weak $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$ decays

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**Abstract** An analysis of  $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$  decay data in a quasi two-body  $B^\pm \rightarrow \pi^\pm [\pi^+ \pi^-]_{S,P,D}$  QCD factorization framework is performed. The short distance amplitudes are calculated including next-to-leading order in  $\alpha_s$  vertex and penguin corrections. The long distance amplitudes due to the  $[\pi^+ \pi^-]_{S,P,D}$  final state interactions are described by the pion non-strange scalar and vector form factors for the  $[\pi^+ \pi^-]_S$   $S$ -wave state and  $[\pi^+ \pi^-]_P$   $P$ -wave state, respectively and by a relativistic Breit-Wigner formula for the  $[\pi^+ \pi^-]_D$   $D$ -wave state. We achieve a good fit of the data with only three parameters for the  $S$  wave, one for the  $P$  wave and one for the  $D$  wave. Our model gives a unified unitary description of the three scalar resonances,  $f_0(600)$ ,  $f_0(980)$  and  $f_0(1400)$  in terms of the pion scalar form factor. We predict for the  $B$  to  $f_2(1270)$  transition form factor,  $F^{Bf_2}(m_\pi^2) = 0.098 \pm 0.007$ .

**Keywords** Hadronic  $B$  decays · QCD factorization · Pion form factors

## 1 Introduction

*Motivations* Three-body hadronic charged  $B$  decays into three charged pions are interesting since they allow i) to study  $CP$  violation, ii) to test the standard model and QCD. The weak and strong phase differences are a measure of the  $CP$  asymmetry  $A_{CP}$ . It can be obtained from the difference between the  $B^+ \rightarrow \pi^+ \pi^- \pi^+$  and  $B^- \rightarrow \pi^- \pi^- \pi^+$  branching fractions. For instance the recent BABAR Collaboration isobar model Dalitz-plot analysis [1] gives for the  $B^\pm$  decay into  $\rho(770)^0 \pi^\pm$ ,

$$A_{CP}[B^\pm \rightarrow \rho(770)^0 \pi^\pm, \rho(770)^0 \rightarrow \pi^+ \pi^-] = (18 \pm 7 \pm 5_{-14}^{+2}) \%. \quad (1)$$

These rare weak decays arise from  $W$ -boson exchange, which, together with the fact that the  $b$  quark mass,  $m_b$ , is large leads, for the short distance amplitude, to a systematic perturbative calculation in a QCD framework. One further assumes factorization which amounts to express the long distance amplitude in terms of form factors [2]. The final state hadronic interactions introduce however important sources of uncertainties.

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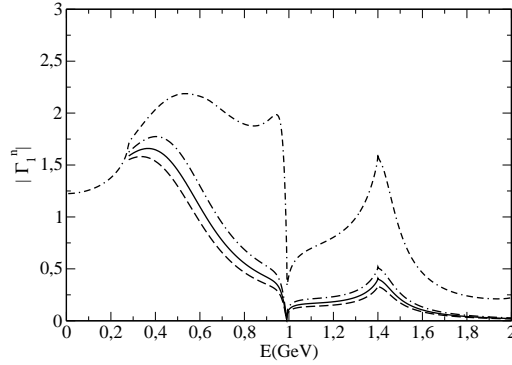
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B. Loiseau · J.-P. Dedonder  
 Laboratoire de Physique Nucléaire et de Hautes Énergies, Groupe Théorie, Université Pierre et Marie Curie  
 et Université Paris-Diderot, IN2P3 et CNRS, 4 place Jussieu, 75252 Paris, France  
 E-mail: loiseau@lpnhe.in2p3.fr

A. Furman  
 ul. Bronowicka 85/26, 30-091 Kraków, Poland

R. Kamiński · L. Leśniak  
 Division of Theoretical Physics, The Henryk Niewodniczański Institute of Nuclear Physics, Polish Academy of  
 Sciences, 31-342 Kraków, Poland



**Fig. 1** Solid line: modulus of the pion scalar form factor  $F_1^{n*}$  from our fit using the NLO effective Wilson coefficients. The parameter  $\kappa$ , ensuring the convergence of our integral equation [8], is 2 GeV. The fitted parameter  $c$ , controlling the high energy behavior, is  $c = (19.5 \pm 4.2) \text{ GeV}^{-4}$ . Double-dash dot line: modulus of the pion scalar form factor calculated in Ref. [11] from the Muskhelishvili-Omnès equations. The dash-dot line and the dashed one represent the variation of the  $F_1^n$  modulus when  $c$  varies within its error band.

*QCD factorization for quasi two-body decays* The  $B$  meson decays into two mesons  $M_1$  and  $M_2$  are well described [3] within the QCD factorization scheme. If no QCD factorization formalism exists for three-body  $B$  decays, one can however use a quasi two-body approach. In the  $B$  rest frame of the three-body  $B^\pm \rightarrow \pi^\pm \pi^- \pi^+$  decays and for low invariant  $\pi^- \pi^+$  mass,  $m_{\pi^- \pi^+} < 2 \text{ GeV}$ , the  $\pi^+$  and  $\pi^-$  mesons of the  $\pi^- \pi^+$  pair move more or less in the same direction. One can then reduce the study of the three-body  $B^\pm \rightarrow \pi^\pm \pi^- \pi^+$  decays to that of quasi two-body  $B^\pm \rightarrow \pi^\pm [\pi^- \pi^+]$  decays provided we make the hypothesis that the states of the  $[\pi^- \pi^+]$  pair originate from a quark-antiquark  $q\bar{q}$  state.

Such an approach has been recently applied by some of us to  $B \rightarrow K \pi^+ \pi^-$  decays by studying  $B \rightarrow K[\pi^+ \pi^-]_{S,P}$  [4, 5] and  $B \rightarrow \pi^\pm [K \pi^\mp]_{S,P}$  [6, 7]. In these works, the long distance  $[\pi^+ \pi^-]_S$   $S$ -wave state includes the scalar  $f_0(600)$  and  $f_0(980)$  resonances [4, 5] while the  $P$ -wave  $[\pi^+ \pi^-]_P$  state [5] takes into account the vector meson  $\rho(770)^0$ . In Refs. [6, 7] the long range  $S$ -wave  $[K \pi^\mp]_S$  state incorporates the scalar  $\kappa(800)$ ,  $K_0^*(1430)$  resonances and the  $P$ -wave  $[K \pi^\mp]_P$  pair the vector meson  $K^*(892)$ .

In the present work we introduce the quasi two-body decays  $B^\pm \rightarrow \pi^\pm [\pi^+ \pi^-]_{S,P,D}$  including, for the long range amplitude in  $[\pi^+ \pi^-]_S$  the scalar  $f_0(600)$ ,  $f_0(980)$ ,  $f_0(1400)$ , in  $[\pi^+ \pi^-]_P$ , the vector  $\rho(770)^0$ ,  $\rho(1450)^0$ ,  $\rho(1700)^0$  and in  $[\pi^+ \pi^-]_D$  the tensor  $f_2(1270)$ . For the short distance amplitudes we shall introduce the leading order (LO) and the next-to-leading order (NLO) vertex and penguin corrections. A full account of the work we present here can be found in Ref. [8].

## 2 Decay amplitudes

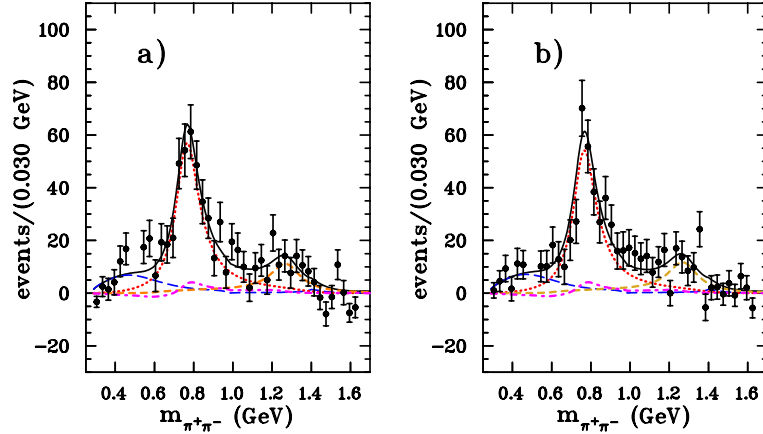
*QCD factorization for  $B^\pm \rightarrow \pi^\pm [\pi^+ \pi^-]_{S,P,D}$*  In the QCD factorization approach one performs a perturbative expansion of decay amplitudes in terms of two small parameters,  $A_{QCD}/m_b$  and  $\alpha_s$ , the strong coupling constant. Using operator product expansion, one writes

$$\langle \pi^\pm [\pi^+ \pi^-]_{S,P,D} | H_{\text{eff}} | B^\pm \rangle = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu) \langle \pi^\pm [\pi^+ \pi^-]_{S,P,D} | O_i(\mu) | B^\pm \rangle, \quad (2)$$

where  $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$  is the Fermi coupling constant and  $H_{\text{eff}}$  is the effective weak Hamiltonian given in Eq. (1) of Ref. [8]. The  $V_{CKM}$  factor arises from product of Cabibbo-Kobayashi-Maskawa matrix elements giving the strength of the coupling of the quarks to the  $W$  boson. The  $O_i(\mu)$  are local operators and the  $C_i(\mu)$  are the short range Wilson coefficients coming from the  $W$  exchange and calculated at the renormalization scale  $\mu$ ,  $\mu \sim m_b$ . Furthermore, schematically, the ansatz of factorization reads,

$$\langle M_1 M_2 | O_i(\mu) | B \rangle = \langle M_1 | j_1 | 0 \rangle \langle M_2 | j_2 | B \rangle [1 + \Sigma_{n=1}^{\infty} r_n \alpha_s^n + O(A_{QCD}/m_b)], \quad (3)$$

where  $j_{1,2}$  are bilinear quark currents. The matrix element  $\langle M_1 | j_1 | 0 \rangle$  is given in terms of the decay constant of the meson  $M_1$  while  $\langle M_2 | j_2 | B \rangle$  can be calculated using the  $B$  to  $M_2$  transition form factor.



**Fig. 2** The light effective  $m_{\pi^+\pi^-}$  distributions from the fit to the BABAR data [1], a) for the  $B^-$  and b) for the  $B^+$  decays. The long-dash line corresponds to the  $S$ -wave, the dot line to the  $P$  wave, the short-dash line to the  $D$  wave and the dot-dash line to the interference contributions. The solid line corresponds to the sum.

At the weak vertices,  $r_n$  represent radiative corrections such as vertex or hard gluon exchanges with the spectator quark contributions [3]. Here the vertex corrections are taken into account but not the divergent hard gluon or the annihilations corrections.

The factorization of the matrix element  $\langle \pi^\pm(p_1)[\pi^+(p_2)\pi^-(p_3)]_{S,P,D} | j_1 \otimes j_2 | B^\pm(p_B) \rangle$  gives two terms. The first term leads to the long distance functions:

$$\langle \pi^\pm(p_1) | j_1 | B^\pm(p_B) \rangle \langle [\pi^+(p_2)\pi^-(p_3)]_{S,P} | j_2 | 0 \rangle \propto F_{0,1}^{B\pi}(q^2) F_{0,1}^{\pi\pi}(q^2), \quad (4)$$

where  $q = p_B - p_1 = p_2 + p_3$ . The  $F_{0,1}^{B\pi}(q^2)$  are the scalar or vector  $B$  to  $\pi$  transition form factor taken from light cone sum rules [9] and the  $F_{0,1}^{\pi\pi}(q^2)$  the scalar or vector pion form factor described below. The second term leads to the long distance terms:

$$\langle [\pi^+(p_2)\pi^-(p_3)]_{S,P,D} | j_1 | B^\pm(p_B) \rangle \langle \pi^\pm(p_1) | j_2 | 0 \rangle \propto G_{R_{S,P,D}\pi^+\pi^-}^n(q^2) \langle R_{S,P,D} | j_1 | B^\pm(p_B) \rangle \langle \pi^\pm | j_2 | 0 \rangle. \quad (5)$$

The vertex functions  $G_{R_{S,P,D}\pi^+\pi^-}^n(q^2)$  describe the resonance  $R_{S,P,D}$  decays into  $\pi^+\pi^-$  pairs. The matrix elements  $\langle R_{S,P,D} | j_1 | B^\pm \rangle$  correspond to the  $B^\pm$  to  $R_{S,P,D}$  transition form factors for which we take  $R_S \equiv f_0(980)$ ,  $R_P \equiv \rho(770)^0$  and  $R_D \equiv f_2(1270)$ . The term  $\langle \pi^\pm | j_2 | 0 \rangle$  is related to the pion decay constant  $f_\pi$ . In our model, we assume the  $G_{R_{S,P}\pi^+\pi^-}^n(q^2)$ ,  $G_{R_D\pi^+\pi^-}^n(q^2)$  vertex functions to be proportional to the scalar or vector pion form factor and to a relativistic Breit-Wigner amplitude [1] for the resonance  $f_2(1270)$ , respectively. From the matrix elements given in Eqs. (4) and (5) one obtains the total symmetrized amplitudes for  $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$  decays as given in Eqs. (21) to (26) of Ref. [8].

*Scalar form factor* The normalized pion scalar form factor,  $\Gamma_1^{n*}(q^2)$ , which is proportional to the form factor  $F_0(q^2) = \sqrt{3/8} \langle [\pi^+\pi^-]_S | u\bar{u} + d\bar{d} | 0 \rangle$ , is calculated from a unitary relativistic coupled channel model including  $\pi\pi$ ,  $K\bar{K}$  and effective  $(2\pi)(2\pi)$  interactions (see Eqs. (27) and (28) of Ref. [8]). The needed  $\pi\pi$  scattering  $T$  matrix corresponds to the solution A of Ref. [10]. The result of our fit (see next section) for the modulus of  $\Gamma_1^{n*}(q^2)$  is plotted in Fig. 1.

*Vector form factor* The pion vector form factor results from the Belle Collaboration analysis [12] of high statistics  $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$  decay data with a Gounaris-Sakurai model for the three vector resonances  $\rho(770)$ ,  $\rho(1450)$  and  $\rho(1700)$ . We use their parameters given in the third column of their Table VII.

### 3 Fit to $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$ decay data

We fit the  $B^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$  BABAR Collaboration data with three parameters for the  $S$  wave, one for the  $P$  wave and one,  $F^{Bf_2}(m_\pi^2)$ , for the  $D$  wave. The  $\chi^2$  fit to the 170 data points from

BABAR's invariant mass distributions together with the experimental branching ratio  $\mathcal{B}_P$  for  $B^\pm \rightarrow \rho(770)^0 \pi^\pm$ ,  $\rho(770)^0 \rightarrow \pi^+ \pi^- = (8.1 \pm 0.5 \pm 1.2^{+0.4}_{-1.1}) \times 10^{-6}$  is  $\chi^2/d.o.f. = 231.6/(171 - 4) = 1.39$ . The fitted strength of the  $S$ -wave amplitude  $\chi_S = -19.4 \pm 2.5 \text{ GeV}^{-1}$ , the two other parameters of the scalar pion form factor being listed in the caption of Fig. 1. The fitted deviation from 1 of the strength of the  $P$ -wave amplitude is  $N_P = 1.122 \pm 0.034$ .

We obtain a good agreement with the experimental  $m_{\pi^+ \pi^-}$  distributions as can be seen in Fig. 2. We find a sizeable contribution of the  $S$  wave just above the  $\pi\pi$  threshold corresponding to the  $f_0(600)$  contribution. The  $f_0(980)$  is not seen as a peak since the pion scalar form factor has a dip near 1 GeV as seen in Fig. 1. The signal for the  $f_0(1400)$  associated with the opening of the effective  $(2\pi)(2\pi)$  channel (see Fig. 1) is tiny. Looking at the distributions (see Ref. [8]) for the cosine of the helicity angle  $> 0$  and  $< 0$ , one observes significant interference contributions, mainly between the  $S$  and  $P$  waves, under the dominant  $\rho(770)^0$  peak.

## 4 Conclusions and outlook

It is quite instructive to see that the quasi two-body QCD factorization allows i) to correlate heavy meson hadronic decay data with other meson-meson interaction data and ii) to give constraints between strong and weak interaction physics. The determination of the strong interaction phases should help in the extraction of the weak angle phase  $\gamma$  or  $\phi_3 = \arg(-\lambda_u^*/\lambda_c^*)$ . Our  $S$ -wave amplitude is proportional to  $\Gamma_1^{n*}(s)$ , its functional form could be used in Dalitz-plot analyses and values can be sent upon request.

Here the data constrain strongly the non-strange pion scalar form factor and in particular its high-energy behavior. It would be interesting if some semi-leptonic process could constraint the pion scalar form factor as in the case of the vector one. On-shell non-strange and strange pion scalar form factors have been used to describe successfully the  $\pi^+ \pi^-$   $S$ -wave final state interaction in  $B \rightarrow K \pi^+ \pi^-$  [4, 5]. One should check if the non-strange pion scalar form factor derived here together with the needed strange component  $\propto \langle [K\bar{K}]_S | s\bar{s} | 0 \rangle$  would yield an equally successful description of the data.

The  $CP$  asymmetry for the  $B$  to  $\rho\pi$  decay is  $A_{CP}[B^\pm \rightarrow \rho(770)^0 \pi^\pm, \rho(770)^0 \rightarrow \pi^+ \pi^-] = 3.6\% \pm 0.2\%$  which is compatible with the experimental value given in Eq. (1). One expects results with smaller errors from the Belle Collaboration, and hopefully, in the near future, from LHCb and from the near term super  $B$  factories.

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